## **Supplementary Materials**

## **A.2 Linearizing the nonlinear constraints**

We linearize *max* function in the risk-related objective (1), using and terms as follows:

|  |  |
| --- | --- |
|  | (A-22) |
|  | (A-23) |
|  | (A-24) |
|  | (A-25) |

Eqs (A-22) and (A-23) calculate the degree of overspeeding in mph, while Eqs (A-24) and (A-25) determine the degree of underspeeding, also in mph. Thus, objective function (1) could be reformed into Eq (A-1.1).

|  |  |
| --- | --- |
|  | (A-1.1) |

Constraints (15) to (17) are nonlinear. Constraints (15) can be rewritten as follows:

|  |  |
| --- | --- |
|  | (A-15.1) |

The product of is nonlinear as it includes a continues variable and an integer variable. To overcome this, an upper bound B is defined for , which is then replaced by a series of binary variables as follows.

|  |  |
| --- | --- |
|  | (A-26) |

Where , , …, are binary variables.

Each product of integers now becomes a product of binaries, and the product of the corresponding binary variables can be easily linearized as follows.

|  |  |
| --- | --- |
|  |  |
|  | (A-26) |
|  | (A-27) |
|  | (A-28) |
|  | (A-29) |
|  | (A-30) |
|  | (A-31) |

Hence, constraint (15) is replaced by:

|  |  |
| --- | --- |
|  | (A-15.2) |

Constraint (16) is linearized in the same way:

|  |  |
| --- | --- |
|  | (A-16.1) |

Again, an upper bound B is defined for , and it is replaced by a series of binary variables :

|  |  |
| --- | --- |
|  |  |
|  | (A-32) |
|  | (A-33) |
|  | (A-34) |
|  | (A-35) |
|  | (A-36) |
|  | (A-37) |

The original constraint (16) is modified to constraint (A-16.2).

|  |  |
| --- | --- |
|  | (A-16.2) |

To linearize Constraint (17), binary variable is introduced along with sets , *LLw*= {0+, , } and *ULw*= {, , 1}, where *w* is the interval for link capacity utilization, represents the set of multiplication factor of *w* for transport mode *m*, *LLw*showsthe lower limit of data rang and *ULw* stands for upper limit of data rang. Constraints (A-38) – (A-41) are then added to the model to linearize constraint (17).

|  |  |
| --- | --- |
|  | (A-38) |
|  | (A-39) |
|  | (A-40) |
|  | (A-41) |

Constraint (A-38) defines the relationship between and for each mode, while Constraint (A-39) ensures that only one value of *w* is selected for each transport mode and link. Constraints (A-40) and (A-41) define the lower and upper bounds of intervals for link capacity utilization to calculate the imbalanced hazmat load risk. The value of *M* is a sufficiently large positive number. As a result, constraints (A-38) – (A-41) replace constraint (17) and linearize the model. The linearized model is as follows:

(MIP)

Objective (A-1.1)

Objective (2)

Subject to:

(3) – (14)

(18) – (21)

(A-22) – (A-41)

(A-15.2)

(A-16.2)